Theory of the $\omega^{-4/3}$ law of the power spectrum in dissipative flows

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It is demonstrated that $\omega^{-4/3}$ law of the power spectrum, with the angular frequency ω in dissipative flows, is produced by the emission of dispersive waves from the antikink of an congested domain. The analytic theory predicts that the spectrum is proportional to ω^{-2} for relatively low frequency and $\omega^{-4/3}$ for high frequency.

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I. INTRODUCTION

Recently, much attention has been attracted to collective dynamics of dissipative particles [1,2]. In particular, physics of granular flows [3–6] and traffic flows [7,8] are developing subjects. In such dissipative flows, we often observe the coexistence of congested regions and dilute regions. It is important to know the mechanism of the congestion of traffic and granular flows. Although we have some exact results on the formation of congested domains in one-dimensional traffic flow [9,10], we still do not understand the details of the fluctuation of dissipative flows.

In experiments of dissipative flows, we usually measure the power spectrum, which is the Fourier transform of the autocorrelation function. It is known that traffic flows and granular flows in a pipe have the power spectra obeying $\omega^{-\beta}$ law with the angular frequency ω [11,12]. Several years ago, Moriyama *et al.* [13] confirmed that granular flow in a pipe should have the spectrum with $\beta=4/3$. We also expect that the power spectrum obeying $\omega^{-4/3}$ law is universal for dissipative flows in the coexistence of congested-flow and sparse-flow [3,13–16]. This law is robust in the experiments of granular flows, which can be observed without tuning of a suitable set of parameters [13,16].

Although the previous papers [3,13] proposed the mechanism of $\omega^{-4/3}$ law, their derivation might be incomplete. We can list several defects in their derivation: (i) They assumed that the system is in a weakly stable region of homogeneous state. However, $\omega^{-4/3}$ can be commonly observed in the case of the coexistence between congestion and sparse flow. The power exponent β is drastically small when there are no definite domains in the systems [15,16]. (ii) The experiments [13,16] suggest that $\omega^{-4/3}$ law is robust without fine-tuning when phase separations take place, but the theory assumes that the system is in the vicinity of the neutral curve of the linear stability analysis. (iii) The theoretical spectrum depends on the wave number, but there is no wave-number dependence in the actual observation in experiments [3,13]. (iv) Although the theory assumes the relaxation process of internal structures, it is not clear what the relevant relaxation process is. Therefore, one is skeptical of the validity of the previous theory to explain $\omega^{-4/3}$ law.

Recently, Takesue *et al.* [17] solved a kink-diffusion problem in the totally asymmetric simple-exclusion process (TASEP) [18] and derived $\omega^{-3/2}$ law of the power spectrum. Although TASEP contains only a kink that connects one congested domain with a dilute region, their analysis is suggestive to understand more realistic situations in traffic and granular flows.

In this paper, we, thus, try to rederive $\omega^{-4/3}$ law in the case of coexistence between congestion and sparse flow. The organization of this paper is as follows. In Sec. II, we explain the main idea of our analysis. In Sec. III, we evaluate the power spectrum of this model. In Sec. IV, we discuss and conclude our results. In the Appendix, we give some explicit expressions of the power spectrum.

II. MAIN IDEA

In order to proceed with the analysis, we should recall that all one-dimensional models for traffic and granular flows in weakly unstable regions can be described by trains of quasisolitons stabilized by small dissipations [3,19–21]. In general, a dilute region is connected with a congested region by asymmetric interfaces [3,20,21], which may be characterized by the soliton equation [19]. We call a front interface the kink and a backward interface the antikink. In general, the antikink (or the kink) exists in the linearly unstable region in the perturbation from the uniform state, and the kink (or the antikink) exists in the linearly stable region [20]. Therefore, the antikink emits dispersive waves backward and they are caught by the next domain. In the simplest situation, we can ignore the widths of the kinks and antikinks, which may be much smaller than the typical domain size.

From the observation of experiments for power spectra, the formation process of domains may not be important, but are important to consider the emission of dispersive waves from an antikink. Thus, we ignore the formation of a congested domain but focus on the decay process of the domain. We also map the model onto a one-dimensional space, where the position fixed in an experimental system is denoted by x, the system size is L, and the boundaries are located at $x=\pm L/2$. For simplicity, we place a detector to measure the power spectrum at $x=0$, i.e., the center of the system. Let us introduce the normalized packing fraction $\phi(x,t) \equiv n(x,t)/n_0(t)$, where $n(x,t)$ and $n_0(t)$ are the density at (x,t) and the saturated density, which depends on time, respectively.

If we assume that an idealistic congested domain exists in the system at time $t=0$, the packing fraction is given by $\phi(x,t=0)=1$ between $x=x_0$ and $x=x_0+l$, and $\phi(x,0)=0$ for otherwise, where l and x_0 are the size of the domain and the position of an antikink at $t=0$, respectively. The equivalent expression is

$$\phi(x,0)=\frac{l}{L}+\sum_{n=1}^{\infty}\frac{\cos\frac{2n\pi x}{L}}{n\pi}\left[\sin\frac{2n\pi(x_0+l)}{L}-\sin\frac{2n\pi x_0}{L}\right]-\sum_{n=1}^{\infty}\frac{\sin\frac{2n\pi x}{L}}{n\pi}\left[\cos\frac{2n\pi(x_0+l)}{L}-\cos\frac{2n\pi x_0}{L}\right]. \quad (1)$$

On the other hand, the antikink is assumed to be unstable because of the dispersion of propagating velocity, though we can ignore such effects for the stable kink. Thus, we assume that the time dependence of $\phi(x,t)$ can be described by

$$\phi(x,t)=\frac{l}{L}+\sum_{n=1}^{\infty}\frac{\cos\frac{2n\pi x}{L}}{n\pi}\left(\sin\frac{2n\pi(x_0+l+c_0t)}{L}-\sin\frac{2n\pi\left\{x_0+c_0t\left[1-\left(\frac{2n\pi\xi}{L}\right)^2\right]\right\}}{L}\right)-\sum_{n=1}^{\infty}\frac{\sin\frac{2n\pi x}{L}}{n\pi}\left(\cos\frac{2n\pi(x_0+l+c_0t)}{L}-\cos\frac{2n\pi\left\{x_0+c_0t\left[1-\left(\frac{2n\pi\xi}{L}\right)^2\right]\right\}}{L}\right), \quad (2)$$

where c_0 is the propagating speed of the kink and ξ is the characteristic length of the dispersion relation. Equation (4) is the expression that the kink whose position is x_0+l+c_0t propagates with the constant speed c_0, while the dispersion of the propagating velocity of the antikink whose position may be $x_0+c_0t[1-(2n\pi\xi/L)^2)]$ makes unable to keep its shape (see Fig. 1). It should be noted that $\int_{-L/2}^{L/2}dx\phi(x,t)$ is not conserved because the phase speed of the antikink is smaller than c_0, but $n_0(t)\int_{-L/2}^{L/2}dx\phi(x,t)$ should be conserved in our picture. However, the correction from $n_0(t)$ is not important because $n_0(t)$, which is determined by the conservation law, causes only the correction of the magnitude of the spectrum.

Thus, the time evolution of $\phi(0,t)$ at the observation point is given by

$$\phi(0,t)=\frac{l}{L}+\sum_{n=1}^{\infty}\frac{1}{n\pi}\left(\sin\frac{2n\pi(x_0+l+c_0t)}{L}-\sin\frac{2n\pi\left\{x_0+c_0t\left[1-\left(\frac{2n\pi\xi}{L}\right)^2\right]\right\}}{L}\right). \quad (3)$$

With the aid of Wiener-Khinchin theorem, the power spectrum $I(\omega)$ and the autocorrelation function $C(t)$ can be written as

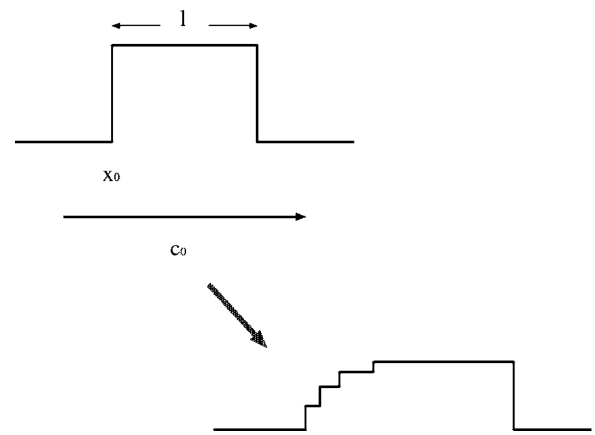

FIG. 1. A schematic picture of the propagation of a domain (its size l) with the speed c_0. The above figure represents an idealistic domain at $t=0$, and the bottom figure represents a decayed domain due to the dispersive wave emitted from the antikink whose initial position is x_0.

$$I(\omega)\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dte^{i\omega t}C(t), \quad C(t)\equiv\langle\phi(0,0)\phi(0,t)\rangle, \quad (4)$$

where the ensemble average in Eq. (4) is interpreted as the average by the initial position of the antikink x_0. Because the domain propagates with c_0 if we neglect the dispersion, the existence probability of domains should be uniform except for the boundary regions. Thus, we may assume the probability distribution function $P(x_0)=1/L$ and $C(t)=1/L\int_{-L/2}^{L/2}dx_0\phi(0,0)\phi(0,t)$.

Before we proceed the analysis, let us summarize critical remarks on our approach. First, the Wiener-Khinchin theorem requires that the system is in a statistically stationary state, but the decay process of a congested domain is not stationary. Since the stationary state is achieved by the supply of particles from the adjacent domain, we need to take into account the import and export of particles between adjacent domains for precise analysis. In addition, the assumption to contain only one domain in a system is unrealistic. Therefore, it may be appropriate to replace the system size L by the average distance between adjacent domains. Nevertheless, our simplification is useful to capture the essence of physical origin of $\omega^{-4/3}$ law.

III. CALCULATION OF THE POWER SPECTRUM

In this section, let us evaluate $C(t)$ and $I(\omega)$. We note that some of expressions are complicated, which are presented in the Appendix.

Substituting Eqs. (1) and (3) into Eq. (4), we obtain

$$C(t)=\frac{l^2}{L^2}+J_0(t)+J_1(t)+J_2(t), \quad (5)$$

where

$$J_0(t) = \sum_{n=1}^{\infty} \frac{1}{2\pi^2 n^2}\left[\cos\frac{2\pi n c_0 t}{L} - \cos\frac{2n\pi}{L}(l + c_0 t)\right], \quad (6)$$

$$J_1(t) = -\sum_{n=1}^{\infty} \frac{1}{2n^2\pi^2}\left\{1 - \cos\left[\frac{2\pi n}{L}c_0 t\right]\cos\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right] - \sin\left[\frac{2\pi n}{L}c_0 t\right]\sin\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right]\right\}, \quad (7)$$

$$J_2(t) = \sum_{n=1}^{\infty} \frac{1}{2n^2\pi^2}\left\{1 - \cos\left[\frac{2\pi n}{L}(l - c_0 t)\right]\cos\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right] + \sin\left[\frac{2\pi n}{L}(l - c_0 t)\right]\sin\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right]\right\}. \quad (8)$$

Here, $J_0(t)$ in Eq. (5) can be calculated as

$$J_0(t) = \frac{l}{2L}\left(1 - \frac{l}{L}\right) - \frac{l c_0 t}{L^2}, \quad (9)$$

where we use the formula $\sum_{n=1}^{\infty} \cos nx/n^2 = \pi^2/6 - \pi x/2 + x^2/4$. Thus, $I_0(\omega) \equiv (1/\sqrt{2\pi})\int_{-\infty}^{\infty} dt e^{i\omega t}[l^2/L^2 + J_0(t)]$ becomes

$$I_0(\omega) = \sqrt{2\pi}\frac{l(L+l)}{2L^2}\delta(\omega) + \sqrt{\frac{2}{\pi}}\frac{l c_0}{L^2}\omega^{-2}. \quad (10)$$

The evaluations of $J_1(t)$ and $J_2(t)$ are nontrivial. When we assume $c_0 t \ll L$, the summation in $J_1(t)$ can be replaced by the integral. From the expansion by $c_0 t/\xi$, we obtain

$$J_1(t) \simeq -\frac{1}{3\pi L}(\xi^2 c_0 t)^{1/3}\left[\int_0^{\infty} dz \frac{1-\cos z}{z^{4/3}} - \left(\frac{c_0 t}{\xi}\right)^{2/3}\int_0^{\infty} dz \frac{\sin z}{z}\right] = -\frac{1}{3L\Gamma(4/3)}(\xi^2 c_0 t)^{1/3} + \frac{c_0 t}{6L}, \quad (11)$$

where we use $\int_0^{\infty} dz(1-\cos z)/z^{4/3} = \pi/\Gamma(4/3)$ and $\int_0^{\infty} dz \sin z/z = \pi/2$ with the Gamma function $\Gamma(z)$. The corresponding Fourier transform of $J_1(t)$ is thus given by

$$I_1(\omega) = \frac{\sqrt{2}}{6\sqrt{\pi}L}(\xi^2 c_0)^{1/3}\omega^{-4/3} - \frac{\sqrt{2}c_0}{6\sqrt{\pi}L}\omega^{-2}. \quad (12)$$

On the other hand, for $l \gg c_0 t$, $J_2(t)$ can be evaluated as

$$J_2(t) \simeq \sum_{n=1}^{\infty} \frac{1}{2\pi^2 n^2}\left\{1 - \cos\frac{2\pi n l}{L}\cos\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right] + \sin\frac{2\pi n l}{L}\sin\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right] - \frac{2\pi n}{L}c_0 t\cos\frac{2\pi n l}{L}\sin\left[\left(\frac{2\pi n}{L}\right)^3 \xi^2 c_0 t\right]\right\} \simeq \frac{l}{\pi L}J_{21}(t) - \frac{c_0 t}{\pi L}J_{22}(t). \quad (13)$$

The explicit expressions for $J_{21}(t)$ and $J_{22}(t)$ are complicated and not important for our purpose (see the Appendix).

In the limit of $\omega \to 0$, $I_2(\omega)$, the Fourier transoform of $J_2(t)$ is dominated by $I_{21}(\omega)$ as

$$I_2(\omega) \simeq \frac{l}{\pi L}I_{21}(\omega) \to -\frac{\sqrt{2}}{6\sqrt{\pi}L}(\xi^2 c_0)^{1/3}\omega^{-4/3}, \quad (14)$$

where $I_{21}(\omega)$ is the Fourier transform of $J_{21}(t)$, and its explicit expression is given by Eq. (A3). It is notable that this asymptotic expression of $I_2(\omega)$ is canceled with the term proportional to $\omega^{-4/3}$ in $I_1(\omega)$. That is, the spectrum obeying $\omega^{-4/3}$ disappears and $I(\omega) \sim \omega^{-2}$ in the limit of $\omega \to 0$.

On the other hand, though $I_{21}(\omega)$ is singular in the limit of $\omega \to \infty$, $I_{21}(\omega)$ is regular for enough large ω. In fact, one can obtain the analytic expansion of $J_{21}(t)$ near $bt = 0.001$ as $2J_{21}(t)/\pi \simeq 1 + \alpha(bt - 0.001) + O[(bt-0.001)^2] \simeq 1 + \alpha bt + \cdots$ with $\alpha = 0.000\,229\,538$. If we replace $J_{21}(t)$ by this approximate function, we obtain the approximate Fourier transform

$$I_{21}(\omega) \sim \frac{\pi^{3/2}}{\sqrt{2}}\delta(\omega) - \sqrt{\frac{\pi}{2}}\alpha b \omega^{-2} \quad (15)$$

for large ω.

Thus, we obtain the power spectrum $I(\omega) = I_0(\omega) + I_1(\omega) + I_2(\omega)$ as

$$I(\omega) = \frac{\sqrt{2}}{6\sqrt{\pi}L}(\xi^2 c_0)^{1/3}\omega^{-4/3} - \frac{\sqrt{2}c_0}{6\sqrt{\pi}L}\left(1 - \frac{6l}{L}\right)\omega^{-2} + \frac{l}{\pi L}I_{21}(\omega) \quad (16)$$

for $\omega \neq 0$. For large ω, $I(\omega)$ is dominated by the term proportional to $\omega^{-4/3}$ as

$$I(\omega) \to \frac{\sqrt{2}}{6\sqrt{\pi}L}(\xi^2 c_0)^{1/3}\omega^{-4/3}. \quad (17)$$

Thus, we derive the spectrum obeying $\omega^{-4/3}$. Figure 2 shows the comparison of Eq. (16) with Eq. (17), where we can see the tail obeying $\omega^{-4/3}$ for large ω, while Eq. (16) seems to obey ω^{-2} for small ω. It is obvious that both expressions (16) and (17) become identical for larger ω.

IV. CONCLUDING REMARKS

It should be noted that the actual process includes many other factors for larger and smaller ω. In experiments, $I(\omega)$ decays exponentially for larger ω because the initial state is not in an idealistic domain, as we have assumed here. To reproduce the full shape of spectrum we need to contain the formation process of domains for our analysis. As stated in the last part of Sec. II, to think of the formation process of domains is also important to justify use of both the Wiener-Khinchin theorem and the Fourier transform itself. Thus, to analyze the process in the balance between the formation and

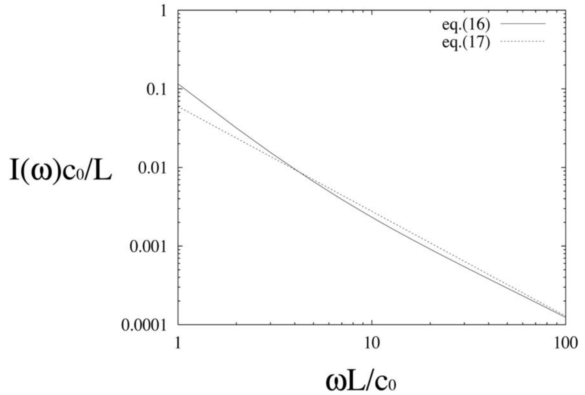

FIG. 2. Log-log plots of Eqs. (16) and (17) as the frequency spectra. We adopt the parameters $l/L=\xi/L=0.3$ and $Lb/c_0=L/l=10/3$.

decay of the domains is an important future problem to be solved. We also indicate that the mutual interaction between domains is important. Thus, it may be appropriate that the system size L we used may be regarded as the average distance between adjacent domains. To include the interaction between domains is also an important point to be improved.

It is interesting that the above defects in our analysis as well as the correction of the magnitude in the observation from $n_0(t)$ are related to the conservation law of macroscopic quantities. It is obvious that these processes should be considered for the precise theory. Nevertheless, we believe that our picture presented here captures the essence of physics and clarifies the mechanism of emergence of $\omega^{-4/3}$ law. This success may be from the fact that $\omega^{-4/3}$ is obtained from the short-time behavior (large ω behavior). Thus, the long-time processes related to the conservation law are not important to obtain $\omega^{-4/3}$ law.

Before we conclude our paper, let us comment on the spectra obeying $1/\omega$-like law in granular flows. For example, Nakahara and Isoda [22] have demonstrated that the behaviors of the power spectrum of granular flows in liquids are different from those in air. In particular, Moriyama *et al.* [23] suggest that $I(\omega)\sim\omega^{-\beta}$ with $\beta=0.95\pm0.05$ for granular flows in the water. The spectra obeying $1/\omega$ law may be from the hydrodynamic effect. Thus, the origin of $1/\omega$ law is completely different from that stated in this paper.

In conclusion, in this paper we have demonstrated that the main process to produce $\omega^{-4/3}$ law is the emission of the dispersive wave from an antikink. This result is universal when isolated congested domains exist in a dissipative flow. Through the analysis, we have revised the previous uncertain picture.

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APPENDIX: SOME EXPLICIT EXPRESSION OF POWER SPECTRUM

In this appendix, we present the explicit expressions that we skipped in Sec. III. Here $J_{21}(t)$ and $J_{22}(t)$ are, respectively, given by [24]

$$\begin{aligned}
J_{21}(t) &\equiv \int_0^\infty dx\frac{1}{x^2}(1-\cos x\cos[x^3bt]+\sin x\sin[x^3bt]) \\
&= \frac{1}{720}\Bigg\{-120\sqrt{3}(bt)^{1/3}\Gamma(-1/3) \\
&\quad \times {}_1F_4\left(-\frac{1}{6};\frac{1}{6},\frac{1}{2},\frac{2}{3},\frac{5}{6};\frac{1}{11\,664(bt)^2}\right) \\
&\quad + 60\sqrt{3}(bt)^{-1/3}\Gamma(1/3)\,{}_1F_4\left(\frac{1}{6};\frac{1}{2},\frac{5}{6},\frac{7}{6},\frac{4}{3};\frac{1}{11\,664(bt)^2}\right) \\
&\quad + \Bigg[120\pi-20\sqrt{3}(bt)^{-2/3}\Gamma(2/3) \\
&\quad \times {}_1F_4\left(\frac{1}{3};\frac{2}{3},\frac{7}{6},\frac{4}{3},\frac{3}{2};\frac{1}{11\,664(bt)^2}\right)+\sqrt{3}(bt)^{-4/3}\Gamma(4/3) \\
&\quad \times {}_1F_4\left(\frac{2}{3};\frac{4}{3},\frac{3}{2},\frac{5}{3},\frac{11}{6};\frac{1}{11\,664(bt)^2}\right)\Bigg]\Bigg\}
\end{aligned} \tag{A1}$$

with $b \equiv \xi^2 c_0/l^3$, and

$$J_{22}(t) \equiv \int_0^\infty dx \frac{\cos x \sin[x^3 bt]}{x} = \frac{1}{216(bt)^{4/3}} \Bigg\{ -18\sqrt{3}\Gamma(2/3)(bt)^{2/3} \times {}_1F_4\left(\frac{1}{3};\frac{1}{2},\frac{2}{3},\frac{7}{6},\frac{4}{3};\frac{1}{11\,664(bt)^2}\right) + \pi\left[36(bt)^{4/3} + \frac{1}{\Gamma(4/3)}\,{}_1F_4 \times \left(\frac{2}{3};\frac{5}{6},\frac{4}{3},\frac{3}{2},\frac{5}{3};\frac{1}{11\,664(bt)^2}\right)\right]\Bigg\}, \quad \text{(A2)}$$

where ${}_1F_4[a_1;b_1,b_2,b_3,b_4;z] \equiv \sum_{k=0}^\infty (a_1)_k z^k/[(b_1)_k \cdots (b_4)_k k!]$ with $(\alpha)_k = \alpha(\alpha+1)\cdots(\alpha+k-1)$ is the generalized hypergeometric function.

The Fourier transforms of $J_{21}(t)$ is given by [24]

$$I_{21}(\omega) = -\frac{\sqrt{\pi/2}}{360 b^{4/3}\omega^{4/3}} \times \Bigg[120 b^{5/3}\,{}_0F_5\left(;\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{5}{6};-\frac{\omega^2}{46\,656 b^2}\right) - 60 b\omega^{2/3}\,{}_0F_5\left(;\frac{1}{2},\frac{2}{3},\frac{5}{6},\frac{7}{6},\frac{4}{3};-\frac{\omega^2}{46\,656 b^2}\right) + 20\sqrt{3} b^{2/3}\omega\,{}_0F_5\left(;\frac{2}{3},\frac{5}{6},\frac{7}{6},\frac{4}{3},\frac{3}{2};-\frac{\omega^2}{46\,656 b^2}\right) - 10 b^{1/3}\omega^{4/3}\,{}_0F_5\left(;\frac{5}{6},\frac{7}{6},\frac{4}{3},\frac{3}{2},\frac{5}{3};-\frac{\omega^2}{46\,656 b^2}\right) + \sqrt{3}\omega^{5/3}\,{}_0F_5\left(;\frac{7}{6},\frac{4}{3},\frac{3}{2},\frac{5}{3},\frac{11}{6};-\frac{\omega^2}{46\,656 b^2}\right)\Bigg]. \quad \text{(A3)}$$

It should be noted that the Fourier transform of $J_{22}(t)$ becomes complex. This is because the expression we obtain depends on the initial condition and the choice of the frame. If the system is Galilei invariant, such a term should be zero. Therefore, we regard the contribution from $J_{22}(t)$ as unimportant.

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